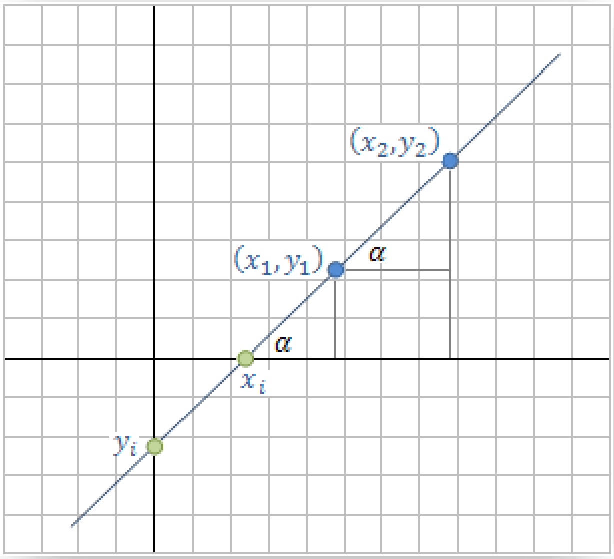


The most general equation of a line is of the form:

$$\boldsymbol{Ax + By + C = 0} \quad (1)$$

where A, B and C are any numbers and A and B are not both zero.

$If\ B \neq 0$ Then dividing by B to obtain the form

$$y = -\frac{A}{B}x - \frac{C}{B} \qquad (2)$$

This is the equation of a line whose slop is:

$$m = -\frac{A}{B} \quad and \quad y_{intercept} = -\frac{C}{B}$$

Equation (2) can be normalized to the general form

$$\boldsymbol{y = mx + b} \qquad (3)$$

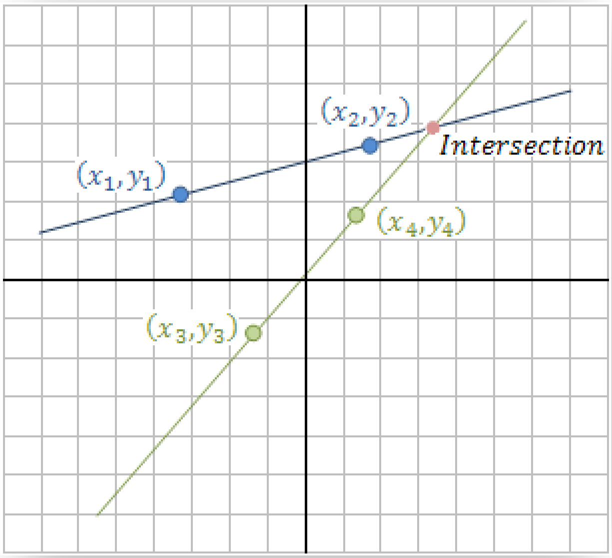
The slop of the line (m) is defined in terms of the inclination and is

$$m = \tan(\alpha) = \frac{y_2 - y_1}{x_2 - x_1} \qquad (4)$$

Note: if the angle α is greater then 90° then the slop is negative

$\alpha(0 \div 90)\ degree \quad : positive\ slop$

$\alpha(90 \div 180)\ degree \quad : negative\ slope$



Necessary condition for two lines to be perpendicular to each other is

that their slopes fulfill the condition

$$m_1 m_2 = -1 \qquad (5)$$

In order to find the intersection point of two lines we have to solve the

system of linear equations representing the lines

$$Ax + By = -C$$

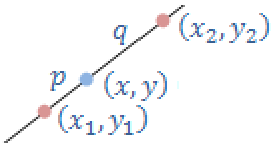
$$Dx + Ey = -F$$

solution by matrix: $x = \frac{\begin{vmatrix} -C & B \\ -F & E \end{vmatrix}}{\begin{vmatrix} A & B \\ D & E \end{vmatrix}} = \frac{BF - CE}{AE - BD}$

$$y = \frac{\begin{vmatrix} A & -C \\ D & -F \end{vmatrix}}{\begin{vmatrix} A & B \\ D & E \end{vmatrix}} = \frac{CD - AF}{AE - BD}$$

If the determinant $\begin{vmatrix} A & B \\ D & E \end{vmatrix} \neq 0$ than intersection point exists.

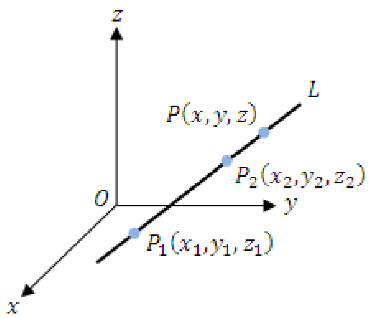
Equation of a line passing through a point (x_1, y_1)	$y = mx + (y_1 - mx_1)$	$y - y_1 = m(x - x_1)$
Equation of a line passing through two points $(x_1, y_1), (x_2, y_2)$	$y = \frac{y_2 - y_1}{x_2 - x_1}x - \frac{x_1(y_2 - y_1)}{x_2 - x_1} + y_1$	$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$
Equation of a line perpendicular to a given slop m *and passing through a point* (x_p, y_p)	$y = -\frac{1}{m}x + \left(y_p + \frac{1}{m}x_p\right)$	$y - y_1 = -\frac{1}{m}(x - x_1)$
Equation of a line perpendicular to a line which is defined by two points (x_1, y_1) *and* (x_2, y_2) *and passing through the point* (x_p, y_p)	$y = \frac{x_1 - x_2}{y_2 - y_1}x + \left(y_p - \frac{x_1 - x_2}{y_2 - y_1}x_p\right)$	$y - y_1 = \frac{x_1 - x_2}{y_2 - y_1}(x - x_1)$
Equation of a line passing through the intercepts $x_i,\ y_i$	$y = -\frac{y_i}{x_i}x + y_i$	$x_i y = -y_i x + x_i y_i$
Equation of a line passing through the point (x_p, y_p) *and parallel to a line which is defined by two points* (x_1, y_1) *and* (x_2, y_2)	$y = \frac{y_2 - y_1}{x_2 - x_1}x + \left(y_p - \frac{y_2 - y_1}{x_2 - x_1}x_p\right)$	$y - y_p = \frac{y_2 - y_1}{x_2 - x_1}(x - x_p)$
Equation of a horizontal line	$y = -\frac{C}{B}$	$y = b$
Equation of a vertical line	$x = -\frac{C}{A}$	$x = a$

Slop (m) *of a line* $y = ax + b$ *or* $y = mx + b$	$m = a$	$m = -\frac{y_i}{x_i}$	$m = \frac{y_2 - y_1}{x_2 - x_1}$
$y_{intercept}$ (y_i)	$y_i = b$	$y_i = -mx_i$	$y_i = y_1 - mx_1$
$x_{intercept}$ (x_i)	$x_i = -\frac{b}{a}$	$x_i = -\frac{y_i}{m}$	$x_i = \frac{y_1 - mx_1}{m}$
$\tan\alpha$	$\tan\alpha = m$	$\tan\alpha = -\frac{y_i}{x_i}$	$\tan\alpha = \frac{y_2 - y_1}{x_2 - x_1}$
Line angle (α) *from x axes (range* $0 \le \alpha < \pi$*)*	$\alpha = \arctan(m)$	$\alpha = \arctan\left(-\frac{y_i}{x_i}\right)$	$\alpha = \arctan\left(\frac{y_2 - y_1}{x_2 - x_1}\right)$
Line midpoint $(\bar{x}, \bar{y})$	–	$\bar{x} = \frac{x_i}{2}$ $\bar{y} = \frac{y_i}{2}$	$\bar{x} = \frac{x_1 + x_2}{2}$ $\bar{y} = \frac{y_1 + y_2}{2}$
Point (x, y) *which divides the line connecting two points* (x_1, y_1) *and* (x_2, y_2) *in the ratio* $p : q$			$x = \frac{px_2 + qx_1}{p + q}$ $y = \frac{py_2 + qy_1}{p + q}$
Angle between two lines (θ)	$\theta = \arctan\left(\left\lvert\frac{m_1 - m_2}{1 + m_1 m_2}\right\rvert\right)$		$\tan\theta = \left\lvert\frac{(y_2 - y_1)(x_4 - x_3) - (y_4 - y_3)(x_2 - x_1)}{(x_2 - x_1)(x_4 - x_3) + (y_2 - y_1)(y_4 - y_3)}\right\rvert$
Angle between two lines given by $Ax + By + C = 0$ *and* $Dx + Ey + F = 0$	$\theta = \arctan\left(\left\lvert\frac{AE - BD}{BE + AD}\right\rvert\right)$		
Slope (M) *of a line perpendicular to a given slop* (m)	$M = -\frac{1}{m}$	$M = \frac{x_i}{y_i}$	$M = \frac{x_1 - x_2}{y_2 - y_1}$

Line form	$Ax + By + C = 0,\quad Dx + Ey + F = 0$	$y = ax + b,\quad y = cx + d$
Distance between two points (D)	$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$	
Distance between intercepts x_i and y_i	$D = \sqrt{x_i{}^2 + y_i{}^2}$	
Distance from a line to the origin	$D = \frac{\lvert C\rvert}{\sqrt{A^2 + B^2}}$	$D = \frac{\lvert b\rvert}{\sqrt{a^2 + 1}}$
Distance from a line given by two points $(x_1, y_1,), (x_2, y_2)$ to the origin	$D = \frac{\lvert(y_2 - y_1)x_1 - (x_2 - x_1)y_1\rvert}{\lvert(x_2 - x_1)^2 + (y_2 - y_1)(x_2 - x_1)\rvert}\sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$	
Distance from a line to the point (x_p, y_p)	$D = \frac{\lvert Ax_p + By_p + C\rvert}{\sqrt{A^2 + B^2}}$	$D = \frac{\lvert ax_p - y_p + b\rvert}{\sqrt{a^2 + 1}}$
Distance from a line given by two points $(x_1, y_1), (x_2, y_2)$ to the point (x_p, y_p)	$D = \frac{\lvert(x_p - x_1)(y_2 - y_1) - (y_p - y_1)(x_2 - x_1)\rvert}{\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$	
Distance between two parallel lines	Parallel condition: $A/B = D/E$ $D = \left\lvert\frac{C}{\sqrt{A^2 + B^2}} - \frac{F}{\sqrt{D^2 + E^2}}\right\rvert$	Parallel condition: $a = c$ $D = \frac{\lvert b - d\rvert}{\sqrt{a^2 + 1}}$ or: $\theta = \tan^{-1} a$ $D = \lvert b - d\rvert \cos\theta$

Intersection point (x, y) of two lines. If $a = c$ or $AE = BD$ then both lines are parallel and no intersection point exists	$x = \frac{BF - CE}{AE - BD}\qquad y = \frac{CD - AF}{AE - BD}$	$x = \frac{d - b}{a - c}\qquad y = \frac{ad - cb}{a - c}$
Intersection point (x, y) of two lines each line defined by two points, first line by the points (x_1, y_1) (x_2, y_2) and second line by (x_3, y_3) (x_4, y_4)	$x = \frac{(x_2y_1 - x_1y_2)(x_4 - x_3) - (x_4y_3 - x_3y_4)(x_2 - x_1)}{(x_2 - x_1)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_1)}$ $y = \frac{(x_2y_1 - x_1y_2)(y_4 - y_3) - (x_4y_3 - x_3y_4)(y_2 - y_1)}{(x_2 - x_1)(y_4 - y_3) - (x_4 - x_3)(y_2 - y_1)}$	

Area of triangle (A) defined by 3 vertices points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$	$A = \frac{1}{2}\lvert[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]\rvert$
Perimeter of triangle (P) defined by 3 vertices points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$	$P = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} + \sqrt{(x_3 - x_2)^2 + (y_3 - y_2)^2} + \sqrt{(x_3 - x_1)^2 + (y_3 - y_1)^2}$

Equation of a plane. ($A, B,$ and C are not all zero)	$Ax + By + Cz + D = 0$
Distance of a point $P_1(x_1, y_1, z_1)$ *from a plane* $Ax + By + Cz + D = 0$	$d = \frac{\lvert Ax_1 + By_1 + Cz_1 + D \rvert}{\sqrt{A^2 + B^2 + C^2}}$
The angle between plane $A_1x + B_1y + C_1z + D_1 = 0$ *and plane* $A_2x + B_2y + C_2z + D_2 = 0$	$\cos\theta = \frac{\lvert A_1A_2 + B_1B_2 + C_1C_2 \rvert}{\sqrt{{A_1}^2 + {B_1}^2 + {C_1}^2}\sqrt{{A_2}^2 + {B_2}^2 + {C_2}^2}}$
Two plans are perpendicular if and only if the attitude numbers A_1, B_1, C_1 *and* A_2, B_2, C_2 *fullfiles the condition.*	$A_1A_2 + B_1B_2 + C_1C_2 = 0$
Line passing through two points $P_1(x_1, y_1, z_1)$ *and* $P_2(x_2, y_2, z_2)$  	*A point* $P(x, y, z)$ *is on* L *if and only if the direction numbers determined by* P *and* P_1 *are propotional to those determined by* P_1 *and* P_2. *If the propotionality constant is* t, *we see that the condition are:* $x - x_1 = t(x_2 - x_1)$, $y - y_1 = t(y_2 - y_1)$, $z - z_1 = t(z_2 - z_1)$ *and the two point form of the parametric equation of a line is:* $x = x_1 + (x_2 - x_1)t$ $y = y_1 + (y_2 - y_1)t$ $z = z_1 + (z_2 - z_1)t$

Equation of a plane passing through 3 points P_1, P_2, P_3. *Since the three points lie in the plane, each of them satisfies the plane equation:* $Ax_1 + By_1 + Cz_1 + D = 0$ $Ax_2 + By_2 + Cz_2 + D = 0$ $Ax_3 + By_3 + Cz_3 + D = 0$	$A = -\begin{vmatrix} 1 & y_1 & z_1 \\ 1 & y_2 & z_2 \\ 1 & y_3 & z_3 \end{vmatrix}$ $B = -\begin{vmatrix} x_1 & 1 & z_1 \\ x_2 & 1 & z_2 \\ x_3 & 1 & z_3 \end{vmatrix}$ $C = -\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ $D = -\begin{vmatrix} x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \\ x_3 & y_3 & z_3 \end{vmatrix}$ $A = -[y_2z_3 - y_3z_2 - y_1(z_3 - z_2) + z_1(y_3 - y_2)]$ $B = -[x_1(z_3 - z_2) - (x_2z_3 - x_3z_2) + z_1(x_2 - x_3)]$ $C = -[x_1(y_2 - y_3) - y_1(x_2 - x_3) + (x_2y_3 - x_3y_2)]$ $D = -[x_1(y_2z_3 - y_3z_2) - y_1(x_2z_3 - x_3z_2) + z_1(x_2y_3 - x_3y_2)]$ *And the plane equation will be:* $\boldsymbol{Ax + By + Cz = D}$
Example: *Find the equation of a plane which passes through the three points:* $P_1(2,3,1)$, $P_2(1,2,-1)$, $P_3(3,-1,2)$	$A = -[1(4 - 1) - 3(2 + 1) + 1(-1 - 2)] = 9$ $B = -[2(2 + 1) - 1(2 + 3) + 1(1 - 3) = 1]$ $C = -[2(2 + 1) - 3(1 - 3) + 1(-1 - 6)] = -5$ $D = -[2(4 - 1) - 3(2 + 3) + 1(-1 - 6)] = 16$ *The plane equation is:* $\boldsymbol{Ax + By + Cz = D}$ $\rightarrow$ $9x + y - 5z = 16$